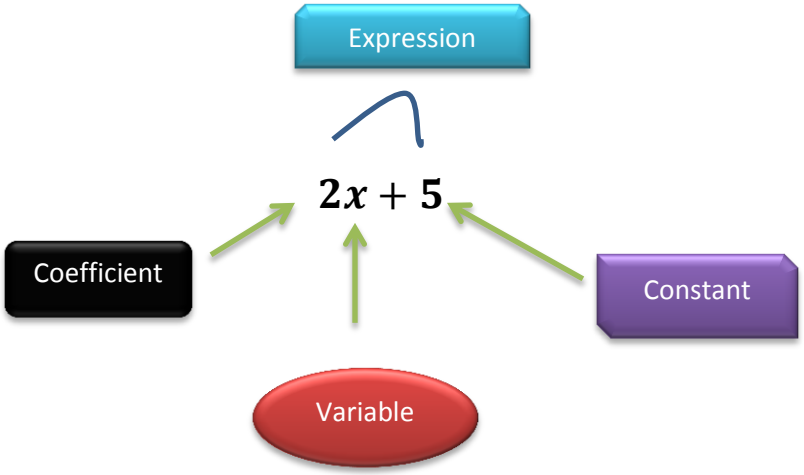
Azim Premji Foundation

Azim Premji Foundation, District Institute, Tonk



Three Days Residential Workshop

24 to 26 September, 2012

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Part I

Introduction

Context

Azim Premji Foundation is working in the Tonk since 2005 and implemented many programmes that seek to demonstrate improvement of quality, build accountability towards quality education among stakeholders and bringing about systemic changes in education delivery system. During these interventions, we supported and worked with teachers at different stages. Over the years working with teachers, whether it is govt. organized training programmes or any workshops, we have developed a robust relationship with them and cultivated an understanding on their academic requirements.

During participation in Govt. training programmes and workshops, it was experienced that important issues and contents of mathematics (Algebra is a matter of so far) were remain untouched. Though teachers manage to deal with Arithmetic somehow, but feel difficulty to clarify the concept and method of Algebra to students in the class room. It led us to develop a course on Algebra and organize a series of workshops with teachers who are teaching Mathematics at middle level in govt. private or any charity (NGO) schools. It was first one in the series in which we covered the basics of Algebra.

Programme and Process: The workshop was conducted as per the agenda prepared earlier. With objectives in view, the discussion was held around the following major topics (**For details please see Annex-1)**:

- History of Algebra
- What is Algebra
- Need, importance and difficulties in linear Algebra
- Basic Math for Algebra
- Introduction to Algebra
- Basic operations with Algebraic expressions and their properties.
- Linear equation in one variable

It was felt that if we come directly to the content, we would not be able to attract participant's attention. Therefore facilitators planned some warm up activities (Inspirational & child songs in morning and games in post lunch session) to takes them ahead. It really worked and ignited the spark among participants for day's activities.

Participants

Around 14 participants, who are teaching mathematics to children at upper primary level, took part in the workshop from different Government and Private schools. **(For detail please see Annex-2)**

Resource Material

The participants were provided with resource materials for reading and references which include the history of Algebra and other, concerning to sessions and activities for Algebra.

Facilitators

- Dinesh Chaudhary
- Hanuman Sharma
- Ashish Gupta
- Vinod Sharma

Supportive Role

- Abhishek Singh Rathore
- Alok Singh Rathore
- Lohit Kumar Joshi
- Vinod Kumar Jain

Guest

- Prasad (Banglore)
- Kushal (Banglore)
- Vinay (Jaipur)

Part II

Proceedings

Welcome & Key note Address

Abhishek, Leader, District Institute, Tonk welcomed the participants and extended greetings. Sharing the purpose of workshop, he described the philosophy of training and its two assumptions as follow;

1. All the class rooms have some problem which can be solved with set of efforts. If they are considered to apply, it is believed that training was successful.
2. Teacher is a sensible human being and can solve all the issues. Due to different circumstances, there cannot be a prescribed solution for them. We own self would find the solution.

"We believe in second one. We would not provide the off the rack solution of any problem even can't. Here, we all would be learning and supporting each other and seeking methods to find out solution of problems and difficulties through discussion and sources for them" Abhishek opined

Further, introducing the Azim Premji Foundation (APF) (origin, vision, mission, interventions and future strategy), he indicated the need of giving attention on three important things to set better education system and stated, how APF is working on them

1. Education of teacher's of teachers
2. Efficacy of people who are leading the system
3. Structure

After welcome and introduction session, Participants were asked to share their class room experiences on Algebra. Most of them were found to be struggling to clarify concept and formulas of Algebra to students. After listening their expectations and issues, the contents, to be discussed in three days, were shared with them. The sessions we could conduct in these three days are as follow

History of Algebra

The session started with the question – *why we need to know the history*? As expected, many interesting answers were derived from participants. Some of them are

- It creates an interest in children
- Gives a knowledge that how one thing came in existence

- How we reach at current system or methodology, who worked, what problems they faced and how they solved them. Knowing these all things can be useful to solve the problems of daily life.
- To know its development, significance and sources of origin

Hanuman and Ashish During History Session

Consolidating the answers, Facilitator went ahead and presented a PPT on History.

Basically two points were covered during history session

1. Developmental Stages in Algebra

The time before Diophantus (around 250): the rhetoric stage

Initially ordinary language was used to describe the solution to problems. No symbols or special designed signs were used. Everything was described in ordinary language.

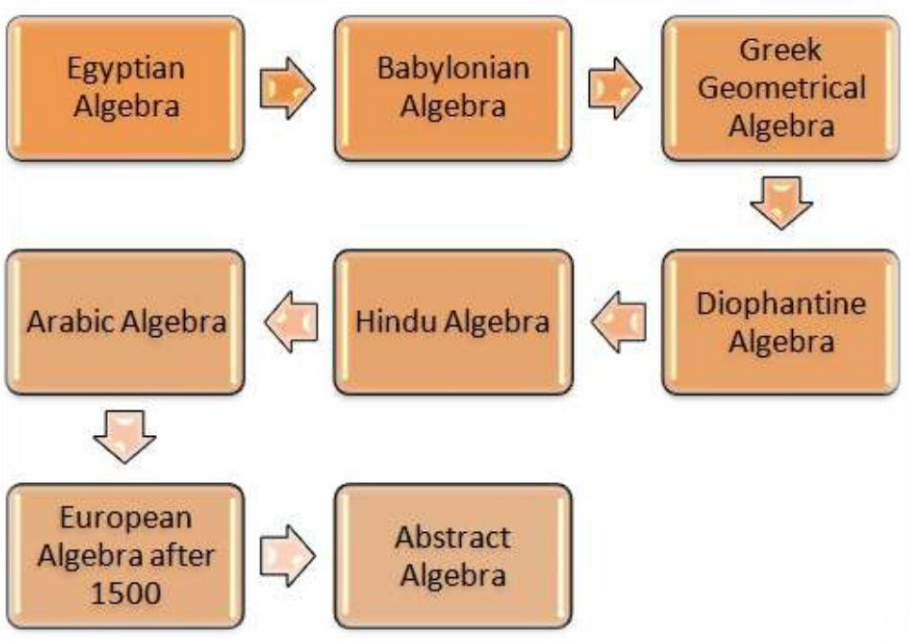
The time from Diophantus (around 250 to 1600): the abbreviation or syncopated stage.

In this period mathematicians started using abbreviations for unknown (but specific) quantities.

The Abstract Stage (1600 – till today): the symbolic stage.

In this period letters were used not only to represent unknowns but letters were also used to represent given quantities (used as parameters).

2. Evolution of Algebra in different civilization



Further he unfolded all the civilizations one by one.

Egyptian Algebra –

"Most of the ancient Egyptian mathematics, including algebra, is based on the Rhind papyrus. This was written about 1650 B.C. and is thought to represent the state of Egyptian mathematics of about 1850 B.C. They could solve problems equivalent to a linear equation in one unknown. Their method was what is now called the "method of false position.

Rhind Mathematical Papyrus

"Their algebra was **rhetorical**, i.e., it used no symbols. Problems were stated and solved verbally." facilitator added.

Through putting below example, he tried to explain the *"method of false position*" and the process of solving problems.

A quantity and its half added together become 16. What is the quantity?

X + 1/2x = 16

Method of False position

Assume

1 = 2

½ = 1

Total 3

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

1	3
2	6
4	12
2/3	2
1/3	1
Total	5⅓

Now 1 = 5⅓

So 2 = 10⅔

Do it thus; the quantity is 10⅔

Half of it	5⅓
Total	16

Babylonian Algebra

Facilitator explained it in below way

- The mathematics of the Old Babylonian Period is (1800 – 1600 B.C.) They had a general procedure equivalent to solving quadratic equations, although they recognized only one root and that had to be positive. In effect, they had the quadratic formula.
- They also dealt with the equivalent of systems of two equations in two unknowns. They considered some problems involving more than two unknowns and a few equivalents to solving equations of higher degree
- There was some use of symbols, but not much.
- Like the Egyptians, their algebra was essentially rhetorical.

- The procedures used to solve problems were taught through examples and no reasons or explanations were given. Like

$$a^2 + b^2 = c^3$$

$$5^2 + 10^2 = 5^3$$

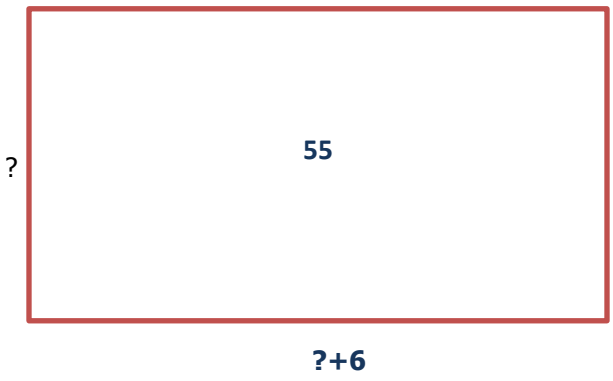
To understand the process of solving problems in this civilization, a video was presented in which problem was solved through geometric methods. It can be seen at login below address.

Example: http://www.youtube.com/watch?v=lr4X5YIjwH4&feature=player_detailpage

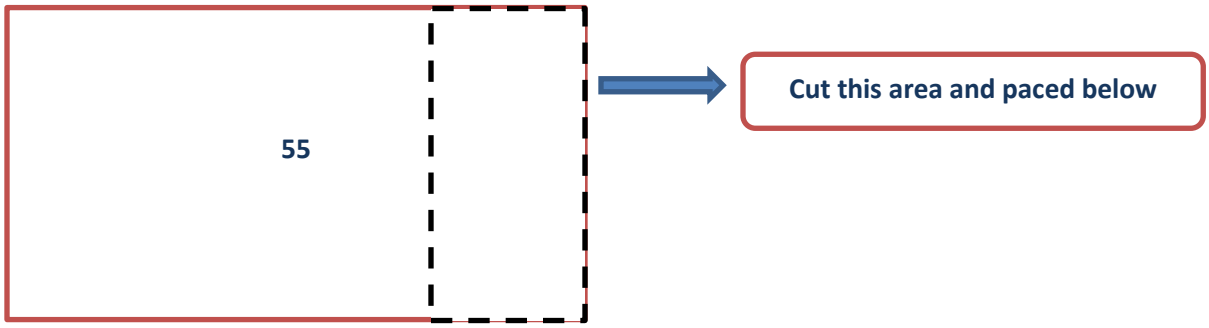
It was in English; hence to understand the problem, it was seen two or three times by participants. But after all, they got it and solved it in geometric methods.

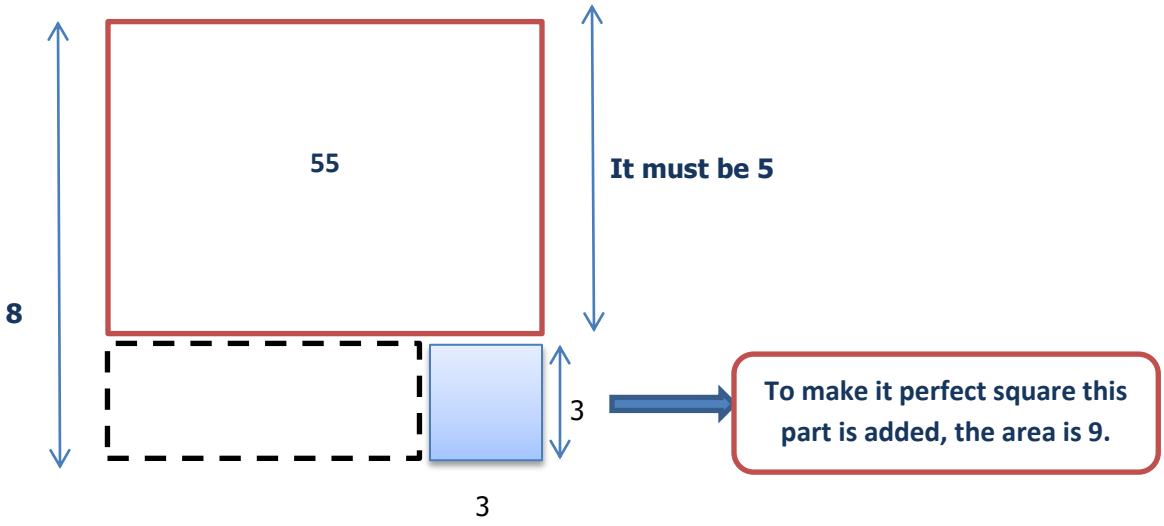
Problem - *The area of the rectangular field is 55 square units. The length is 6 units more than its width. How long its width.*

First, present the problem in geometric form, like below



Converted the rectangle in perfect square





Now it has become a perfect square. Total area is 55 + 9 = 64 square meter. One side = 8.

X = 5, so length in 5+6 = 11.

Greek Geometrical Algebra


DIOPHANTI
ALEXANDRINI
ARITHMETICORVM
LIBRI SEX.
ET DE NVMERIS MVLTANGVLIS
LIBER VNVS.
AVCTORE CLAVDIO GASPARE BACHETO
LVTETIAE PARISIORVM,
Sumptibus SEBASTIANI CRAMOISY, via
Iacobea, sub Ciconiis.
M. DC. XXI.
CVM PRIVILEGIO REGIS

The Greeks of the classical period, who did not recognize the existence of irrational numbers, avoided the problem thus created by representing quantities as geometrical magnitudes. Various algebraic identities and constructions equivalent to the solution of quadratic equations were expressed and proven in geometric form. In content there was little beyond what the Babylonians had done, and because of its form geometrical algebra was of little practical value. This approach retarded progress in algebra for several centuries. In the 3rd century, the Greek mathematician *Diophantus of Alexandria* wrote his book Arithmetica.

Diophantine Algebra

He briefed the important points.

- He introduced the **syncopated** style of writing equations. The rhetorical style remained in common use for many more centuries to come.
- He gives a treatment of indeterminate equations - usually two or more equations in several variables that have an infinite number of rational solutions. Such equations are known today as "Diophantine equations".
- He had no general methods. Each problem in the Arithmetic is solved by a different method.
- When a quadratic equation had two positive rational roots he gave only one as the solution. There was no deductive structure to his work.

$ax + by = 1$ **This is a linear Diophantine equation**

Example: **Problems of Diophantus**

Solutions in whole numbers or fractions:

Problem $2x+3y = 10$

Sol. $X = 2, y = 2$

Hindu Algebra

"They invented and treated zero as a number and discussed operations involving this number" facilitator reiterated. Concept of Zero was discussed in detail and participants sought the answers of many questions like, *why it is invented, how we felt the need of Zero* and so on

"To clear the concept of nothing, it treated as a number" facilitator responded. He put some examples to clarify this.

Before existence of zero 403 was written like 4 3. They left some space between numbers. After some time they started to use symbols such as 4.3 or 4@3. But question aroused that will another person also get it in same meaning.

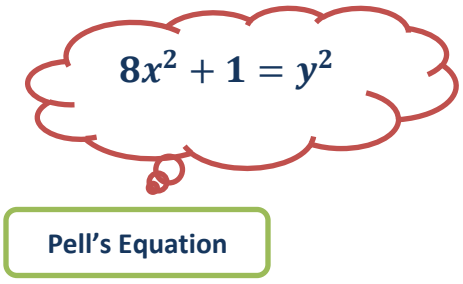
$x + 3 = 3$, **What is the value of x?**

So there is big contribution of 0 and debt numbers in algebra.

"They introduced negative numbers to represent debts. The first known use is by Brahmagupta about 628. Bhaskara (1114 AD) recognized that a positive number has two square roots. The Hindus also developed correct procedures for operating with irrational numbers" he added and clarified it with example.

Example: 'Pell's Equation' - Bhaskara (1100 AD)

Problem- what is that square which multiplied by 8 becomes- together with unity- a square?



Later facilitator conferred on Hindu Mathematician ***Shridharacharya*** and his formula to find the roots of a quadratic equation of form

$ax^2 + bx + c = 0$ **This is as follows-**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} ; a \neq 0$$

For proof: click to this link
http://www.eklavya.in/pdfs/Sandarbh_76/65-68_Where_is_the_Square_to_Complete.pdf

He elaborated the whole process that how it was derived.

Two third time of the day was gone away in discussion on above points. The process of deriving method to find the roots of a quadratic equation, left the participants little bit tired mentally. Therefore the discussion on remaining part of history was shifted on next day. At last one hour, Dinesh took the charge and facilitated another topic, *what is Algebra.*

In this session participants were divided in group and managed to recall their knowledge and understanding over algebra. Participants shared their views to each other and prepared chart presentations till end of the day.

2nd Day
25 September, 2012

It was 9:00 AM in the morning, the scheduled time for the participants to arrive but only few of them were present. Facilitator made a small circle with those who were present to keep them busy and chanted some Chetna and Baal Geets while others arrive.

After 15 minutes of warm up session, he guided the participants for recap of first day. He designed the session in question answer form which not only summarized the last day's actions and discussions but also provided us the feedback and suggestions. Below three questions were asked to participants.

- What did yesterday?
- What went well?
- What to be added or improved?

Most of them were applauded the first session History of Algebra. "It was first in my life that I am going through with history of Algebra" participant pointed out. They also enjoyed the geometric methods for solving the Algebraic problems.

Facilitator started the session ……..

Once again Ashish initiated the day and elaborated remaining part of history

Arabic Algebra

First he showed the cover page of book Hisab aljabrw'al muqabala written by mathematician Mohammed ibn-Musa al-Khwarizmi and explained below points

- They took over and improved the Hindu number symbols and the idea of positional notation.
- Like the Hindus, the Arabs worked freely with irrationals.
- The algebra of the Arabs was entirely **rhetorical**.
- This title is sometimes translated as "Restoring and Simplification" or as "Transposition and Cancellation."

Cover Page of book

Facilitator added

- In algebra the Arabs contributed first of all the name. The word "algebra" comes from the title of a text book.

- Equation is called Aljabrw'al in Arabi and muqabala in Pharsi. Brhamgupt called is ***Beejganit*** first time.

"They could solve quadratic equations, recognizing two solutions, possibly irrational, but usually rejected negative solutions" Ashish added and explained it with example.

Problem - *sum of square and 10 roots are equal to* 39 *units.*

Facilitator added

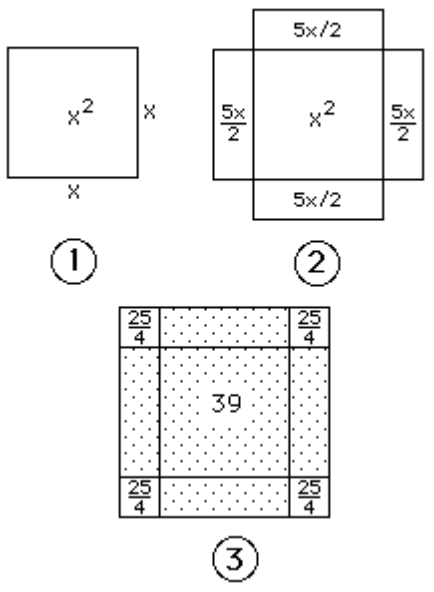
This problem was mentioned by Al-khwarizmi (Mathematician) in his book. Whatever that may be the problems, he (Al-khwarizmi) also solved them through geometrical methods

He further asked to participants for solving the problem in geometrical method. It was not tough for them as they have solved the problem earlier through this method. Later facilitator shared process of Al-khwarizmi in which he solved the problem.

Equation: $x^2 + 10x = 39$

- Al-Khwarizmi starts with a square of side x, which therefore represents x^2 (Figure 1).
- To the square we must add $10x$ and this is done by adding four rectangles each of breadth 10/4 and length x to the square (Figure 2).
- Figure 2 has area $x^2 + 10x$ which is equal to 39.
- We now complete the square by adding the four little squares each of area. 5/2x5/2 = 25/4.
- Hence the outside square in Fig 3 has area 4 x 25/4 + 39 = 25 + 39 = 64.
- The side of the square is therefore 8.
- But the side is of length 5/2 + x + 5/2 so $x + 5 = 8$, giving $x = 3$.

al-Khwarizmi completes the square



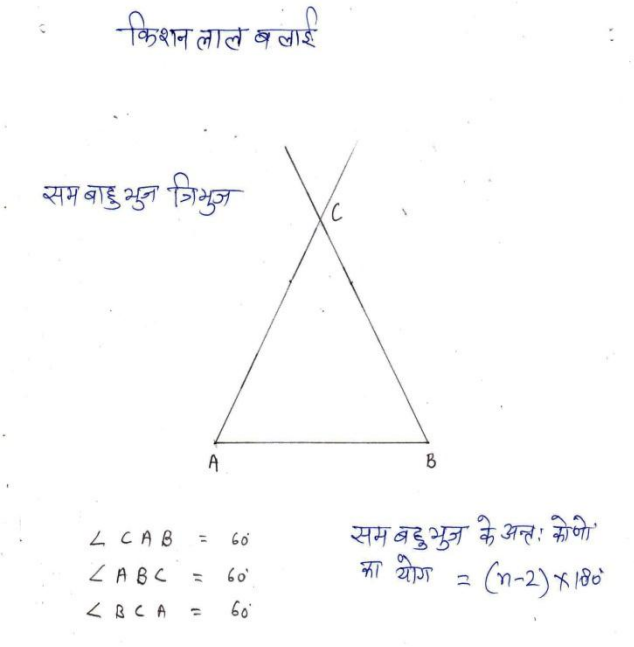
Thus, facilitators introduced the different civilization and methods of solving problems. These examples created curiosity among participants. It covered the almost one day but we never felt that participants are getting bored.

Three Impossible Classical Constructions

Ashish referred on these problems little bit on first day during history session and promised to participants for sharing these problems. Now he shared them and encouraged for trying to solve them.

1. Squaring the Circle. Given a circle, construct a square that has exactly the same area as the circle.

2. Trisecting an Angle. Given an angle, construct an angle whose measure is exactly 1/3 the measure of the original angle.



Participant's Attempt

3. Doubling a Cube. Given the length of the side of a (three-dimensional) cube, construct a length so that a cube with an edge of this length will have exactly double the area of the original cube.

Participants made attempt to solve the second one but could not do *(see above picture*). Though some of them put a very decent reason that why they are impossible. Such as

"First one can be solved if we could find the exact value of π (Pie), that is not possible" participant assumed

To maintain the impetus and keep on their interest, facilitator (Vinod) put some puzzled to participants as one of us suggested during last review meeting. It really worked and involved all attendants completely even they took relief after solving them.

Continue to what is Algebra and introduction to Algebra -

A short discussion on it had been done at first day. They had discussed it in group and prepared presentation on charts. Now one person from each group came on center and presented whatever they discussed in group.

Groups presented –

- It is branch of mathematics in which we create relation among mathematical numbers.
- Mode of writing the numbers in symbols.
- Mode of solving mathematical problems through symbols
- To establish relation between variable and constant
- It helps in solving problems through generalization
- It is branch of mathematics through which we find out the value of unknown
- In which we use algebraic expressions to find out value of unknown.

These are the some points which came from the participants. Getting views of participants, Hanuman led the session. Before going to define Algebra, he introduced Arithmetic and its relation to Algebra.

"*Arithmetic* is a branch of mathematics that deals with properties of the counting, and also whole numbers and fractions and the basic operations applied to these numbers" he defined. In this we go to the generality from specific. Same thing we do in Algebra" he added.

After an hour of brain storming and discussion, consensus made in below points

- Algebra is the part of mathematics in which letters and other symbols (x, y, z or a, b, c) are used to represent numbers and quantities in formulae and equations.
- It helped to solve the problems easily. *(What is the square which combined with ten of its roots will give a sum total of 39. Making equation in algebra it can be solved easily. (* $x^2 + 10x = 39.$ *)*
- IT is a generalize form of Arithmetic.

He depicted many examples and convinced them in lucid language to prove and clear the above points.

Pre-Algebra

Facilitator pitched a question to participants.

"Why Algebra starts from class 6th or what a child should learn before dealing with Algebra"

Participants responded

- Complete knowledge for operations (Addition, Subtraction, Multiplication and Division)
- Mathematical Formulation of problems
- Understanding of the properties of numbers
- The knowledge of Natural, whole and Q numbers.
- Simple and Decimal fractions
- Knowledge of rational numbers

Facilitator added…..

"Above are the primary concepts to understand the Algebra. If a respectable work has been done on above points then a students can easily deal with algebraic problems"

To understand the above concepts in depth, facilitator again asked the series of questions and invited the participants one by one to solve it.

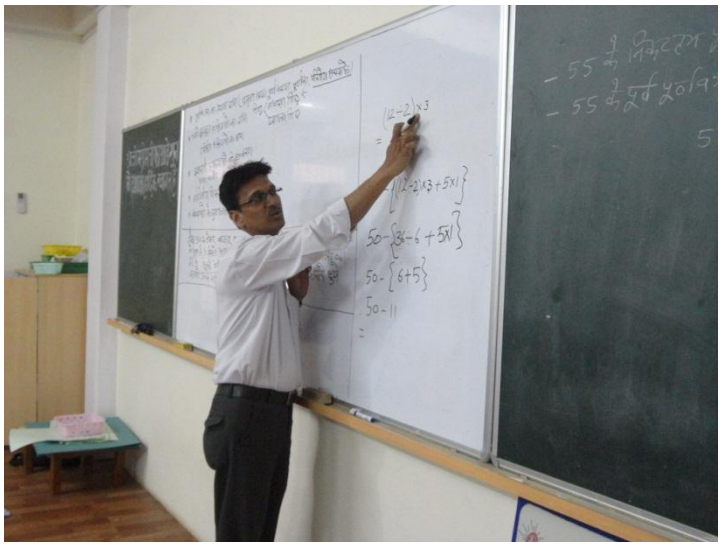
Question related to order of operations.

रमेश 50 रूपये लेकर बाजार गया। उसने सब्जी वाले से 12 रूपये के 2किलो के भाव से 3 किलो आलू खरीदे। तथा फल वाले से 5 रूपये प्रति किलो की दर से 1 किलो अमरूद खरीदे। संक्रियाओं का क्रम तय करते हुये बताओ उसके पास अब कितने रूपये शेष रहे। ।

Participant's attempt

Mathematical Formulation - $\mathbf{50 - \{(12 \div 2) \times 3 + 5 \times 1\}}$

Solution - $\mathbf{50 - \{(6) \times 3 + 5 \times 1\},}$



$$\mathbf{50-\{18+5\},50-23=27}$$

Question 2 $$\mathbf{1\frac{1}{2}+\left[1\frac{1}{4}-\left\{1\frac{1}{2}-\left(1\frac{1}{4}-\overline{1\frac{1}{2}-1\frac{1}{4}}\right)\right\}\right]}$$

Participants' attempts

$$\mathbf{\frac{3}{2}+[\frac{5}{4}-\left\{\frac{3}{2}-\left(\frac{5}{4}-\frac{3}{2}+\frac{5}{4}\right)\right\}]}$$

Removed Bar

$$\mathbf{\frac{3}{2}+[\frac{5}{4}-\left\{\frac{3}{2}-\frac{5}{4}+\frac{3}{2}-\frac{5}{4}\right\}]}$$

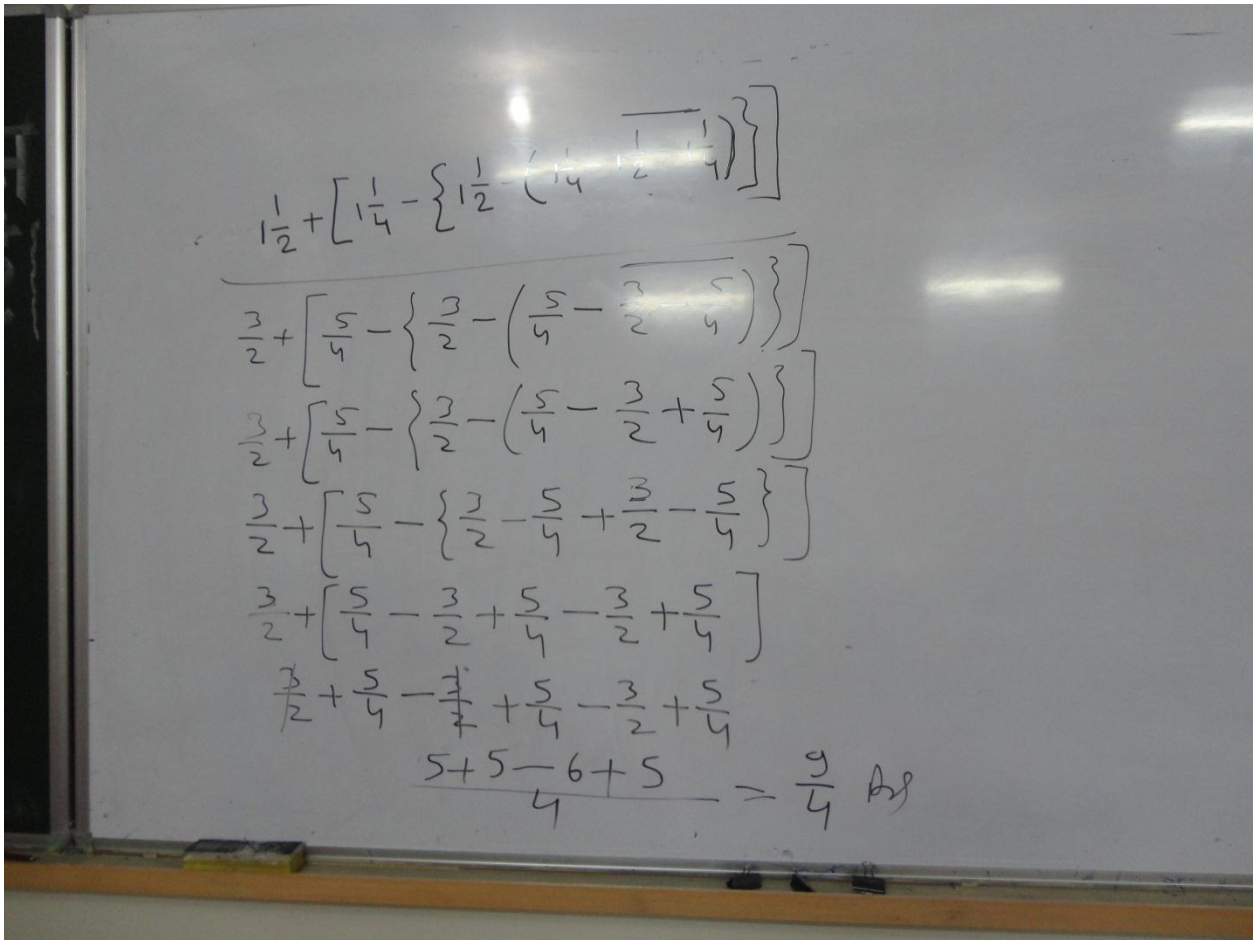
Removed Small Brackets in changing order of operations

$$\mathbf{\frac{3}{2}+[\frac{5}{4}-\frac{3}{2}+\frac{5}{4}-\frac{3}{2}+\frac{5}{4}]}$$

Removed Curly Brackets in changing order of operations

$$\frac{3}{2}+\frac{5}{4}-\frac{3}{2}+\frac{5}{4}-\frac{3}{2}+\frac{5}{4}$$

Removed Square Brackets

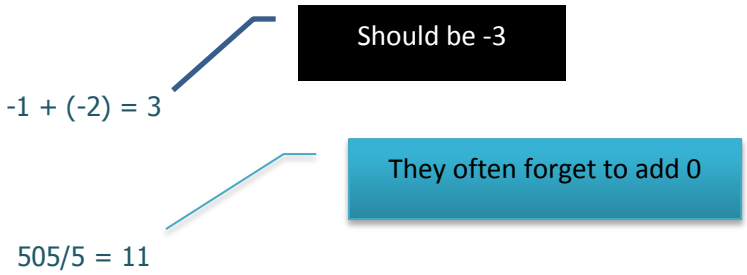


Participant's attempt

$$\frac{3}{2}+\frac{5}{4}-\frac{3}{2}+\frac{5}{4}-\frac{3}{2}+\frac{5}{4}=\frac{9}{4}$$

Participants intervened

"They (students) also often make mistake to complete the operations in integer numbers" Such as



Here facilitator introduced some innovative practices to teach the operations and clarifying concept of adding 0. He quoted some examples related to integer numbers

Question related to integer numbers

1- $\mathbf{4+2-3+7=?}$
2- $\mathbf{4-2-3+7=?}$
3- $\mathbf{4+2-3-7=?}$
4- $\mathbf{4+2+3-7=?}$
5- $-(-4-2)+3+7=?$

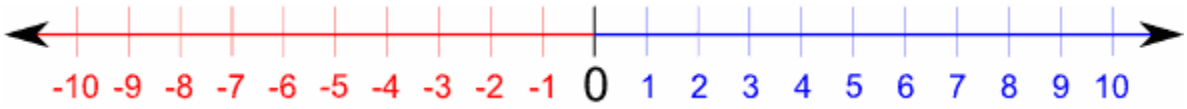
Participant's remark

These all questions will be solved through number line.

Participants drew the line and stated their way to solve these types of questions or to teach children in class rooms.

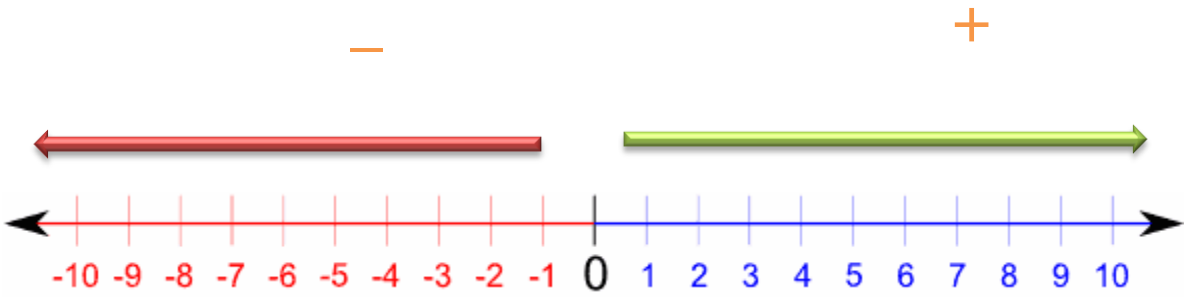
Facilitator added

Positive numbers are right hand of number line and negative numbers are in left hand.

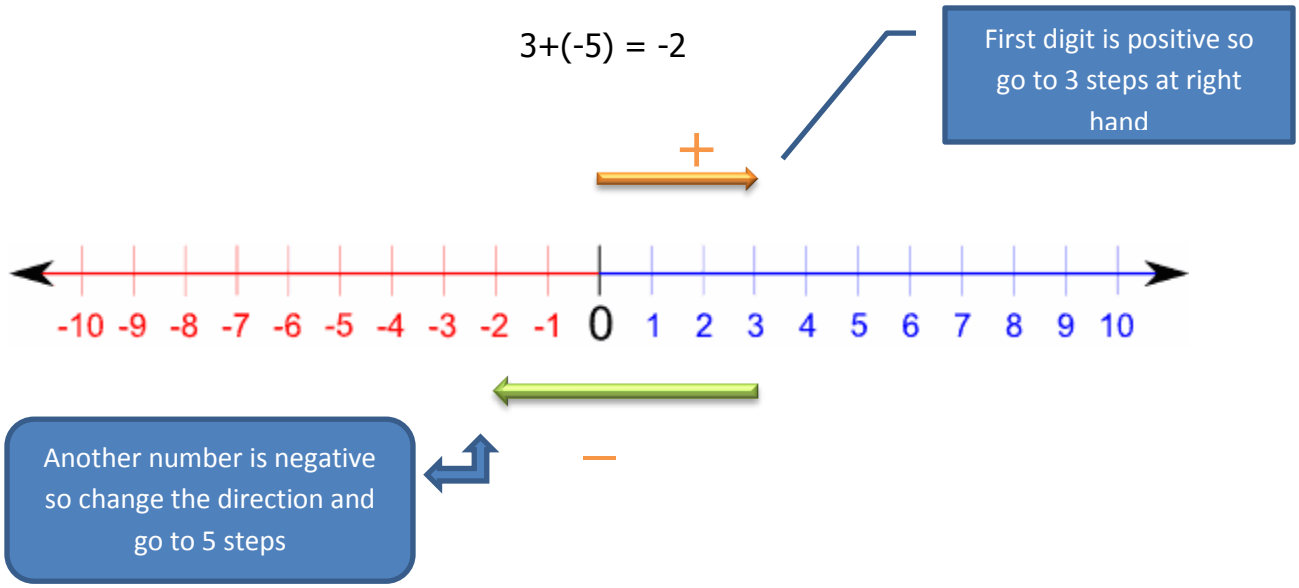


When the number is positive, count to the right.

When the number is negative, count to the left.



Example



The sound discussion was held on plus minus calculation and the way of teaching to solve these types of questions.

Through these examples, facilitator clarified required primary level of understanding and their important to deal with Algebra. The long discussion was held on the concept of removing brackets with changing order of operation in which students often make mistakes.

Analysis of Text books

Once again participants requested to sit in their respective groups and guided for another group activity. All three groups were given set of chapters from books of different publication (NCERT, SIERT and Private publication) and asked to analyzed them on the basis of below points

- Approach towards unknown and variable.
- Association with foreknowledge
- Discussion on importance of learning
- Inclusion of activities
- Initiation and steps of proceedings
- Engagement with atmospheric experiences.

First, all groups were required to do comparative analysis among them, afterwards to present review of one publication, was assigned to them as follow

- Group I – Private Publication
- Group II – SIERT
- Group III – NCERT

After half an hour of discussion, one or two members of each group presented the given task.

Group I – Private Publication

- It explained the concept of unknown through examples but generalization is not clear.
- With the help of properties, it showed the relationship with Arithmetic.
- It tries to convince Algebra through letters.
- Operations can be done with letters, has been explained it through examples.
- The development of the lessons is very good but use of properties in beginning seems irrelevant.

Group II – SIERT

- It uses signs for unknown. For instance, *the diameter of a circle is twice to its radius*. They composed it as follows

$$D = 2r$$

D = Diameter and r = Radius

- Approaches are not so apparent.
- Not defined the variable and constant.

Group III – NCERT

- It introduced different branches of mathematics and relation among them
- Also explained history of Algebra little bit.
- It introduced Algebra through activities (See the example of making L by matchsticks)

Through this session, all the participants made an understanding on what and how should be taught to children in beginning of Algebra and identified the suitable route and activities to make it comfortable for children.

At last, facilitator passed two puzzles to participants and asked to come with solution at next day.

Puzzle -

कहानी – राजा के घोड़े

एक राजा था। वह केवल 9 तक गिन सकता था। आप किस संख्या तक गिन सकते हैं?

राजा को घोड़ों से बहुत प्यार था। लेकिन वह कभी भी उन सभी को नहीं गिन पाता था। वह उन्हें इस तरीके से बाँधता था कि उसे हर तरफ़ से केवल 9 तक गिनने की ज़रूरत हो।

आप बता सकते हैं –
राजा के पास कुल मिलाकर
कितने घोड़े थे? .....

एक दिन अपने चार घोड़ों के साथ एक यात्री वहाँ आया। रात बहुत हो गई थी इसलिए वह रात को वहाँ रुकना चाहता था। रखवाला घबराकर बोला– ना बाबा ना, राजा को पता चल गया तो बस हो गई छुट्टी। यात्री ने उससे कहा– आप घबराओ नहीं राजा को कभी पता नहीं चल पाएगा। यात्री ने घोड़ों को इस तरह रखा।

अब आप बताइए यहाँ कितने घोड़े हैं? .....

रात को राजा वहाँ आया और उसने घोड़ों को गिना। चारों तरफ़ से उसने 9 ही गिने। उसने कहा – हाँ सब ठीक है और घोड़े गिनकर वह खुशी से सोने चला गया।

सुबह उठते ही चालाक यात्री ने एक दूसरी तरकीब लगाई। उसने अपने चार घोड़े तो बाहर निकाल ही लिए साथ ही वह राजा के कुछ घोड़े भी साथ लेकर चला गया। वह राजा के घोड़ों को इस तरह से खड़ा कर गया।

मूर्ख राजा को यह पता नहीं लगा कि उसके कुछ घोड़े वहाँ हैं ही नहीं क्या आप राजा को बता सकते हैं?

अब आप बता सकते हैं –

वहाँ पर कुल मिलाकर कितने घोड़े शेष बचे हैं? .....

राजा के कितने घोड़े यात्री चुराकर ले गया? .....

3rd Day

26 September, 2012

Day began with songs and recap. Then we came to the puzzles, were asked at the end of last day. It was great to see; many of them worked at home and came with solution. They came on board and solved them more than one way. In next few minutes, participants shared the process, discussed (mostly on king and horses, see above two pictures) and made a common understanding on the puzzle.

Nature of Algebra

Coming to the agenda of last day, facilitator started the session with a question.

Question – *माँ अपने तीन बच्चों के लिए इडलिया बनाकर बराबर बांटकर खाने को कह कर चली गई है। लेकिन तीनो बच्चे अलग-अलग समय आए और तीन हिस्सों में बांटकर अपना-अपना हिस्सा खाकर चले गए। इस कारण अंत में कुछ इडलिया फिर बच गई जिन्हें मां ने बच्चों में बराबर बांटकर पूरा किया। तो बताओं मां ने कम से कम कितनी इडलिया बनाई होंगी।*

Through this example, he tried to convince them the nature of Algebra. For instance, question says that

- Idlees (*इडलिया)* to be divided in three parts.
- Every child should get equal portion.
- One of them comes, divides them in three parts, eats its part, and mix remaining both the parts. Then second one came, does same thing. Afterwards third one and their mother repeat the process.

Ashish added……..

"Here we need to think in different direction (mentioned above), so Algebra teaches us to *generalize the things"*

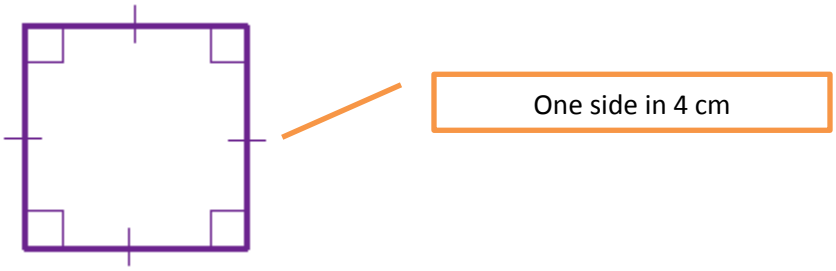
He further described the three key steps on which someone should work in solving process any problem from beginning to end. These are

- Specific to Generalization (getting rules or formula for solution)

- Generalization to Specific (Symbolic form)
- Simplification

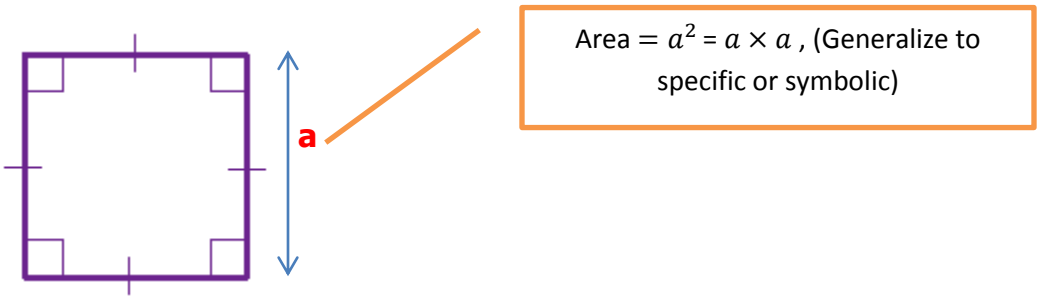
He clarified this explaining below example

Make a square



Explained the formula to get area of square

- All sides are equal so the Area is the **side length squared**:(*Specific to Generalize*)



So, $4 \times 4 = 16$ $(Simplification)$

Facilitator added……..

- First, we should teach generalization to children
- It can be clarified through getting examples of their surroundings.

"But how we should teach the generalization and concept of variable and unknown to students" facilitator put another question and invited participants to share their methods of teaching in class room.

Participants relied…….

Through examples,

Participants - 1

Problem - What we should add in 3, it becomes 8

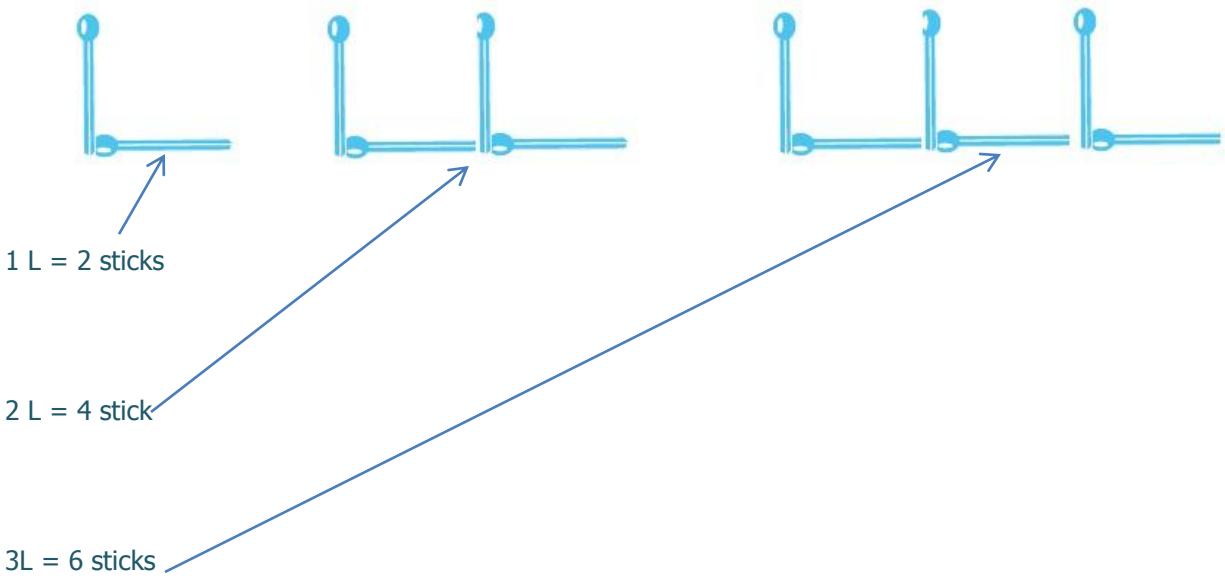
OR

$\Delta + 39 = 67,$ *what is value of* Δ

It is variable

Participant – 2

Make an **L** form Match Sticks



Children go on forming the pattern with 1 L, 2Ls and 3Ls and so on and prepare a table

No. of Ls formed	1	2	3	4	5	6	7	8	9
No. of Match sticks required	2	4	6	8	10	12	14	16	18

While writing the table, now students realize that the no. of match sticks required is twice the no. of Ls formed.

Number of matchsticks required = $2 \times number\ of\ Ls$

Now if I asked, how many matchsticks are required to form 100Ls, child will easily give answer – 200. Because he has understand, here n is the no. of Ls in the pattern and n takes value 1, 2, 3, 4…….. The value n of goes on changing (increasing), as a result, the no. of matchsticks required also goes on changing (increasing).

N is an example of variable.

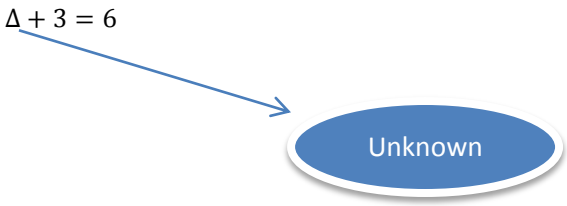
Thus, one or two more participants came and shared the way of clarifying the concept of variable to children.

Now question aroused, are unknown and variable have same meaning?

Facilitator Added……

Both are different word and have different meaning.

- Variable means something that can vary, i.e. change which has been described above (matchsticks and L) in detail.
- Unknown means something which need to be found.



Participants asked**……..**

- What is the most suitable method to clarify the concept of variable and unknown?

Facilitator added……..

- The second examples, making L from matchsticks seems suitable as in which children, learning by doing.
- NCERT also saying like this in its class 6th book.
- It talks about mere variable (not unknown). But we can clear the concept of variable and unknown, even difference.

Algebraic Expression

Again facilitator started with the problem

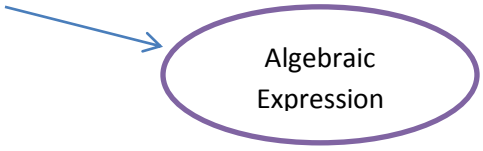
"Before 5 year, the age of Shyam was twice of Ram. What is the age of Ram?"

Suppose,

The age of Ram = x years

Before 5 years, the age of Shyam was twice of Ram = $2x$

Then the age of Shyam = $2x + 5$

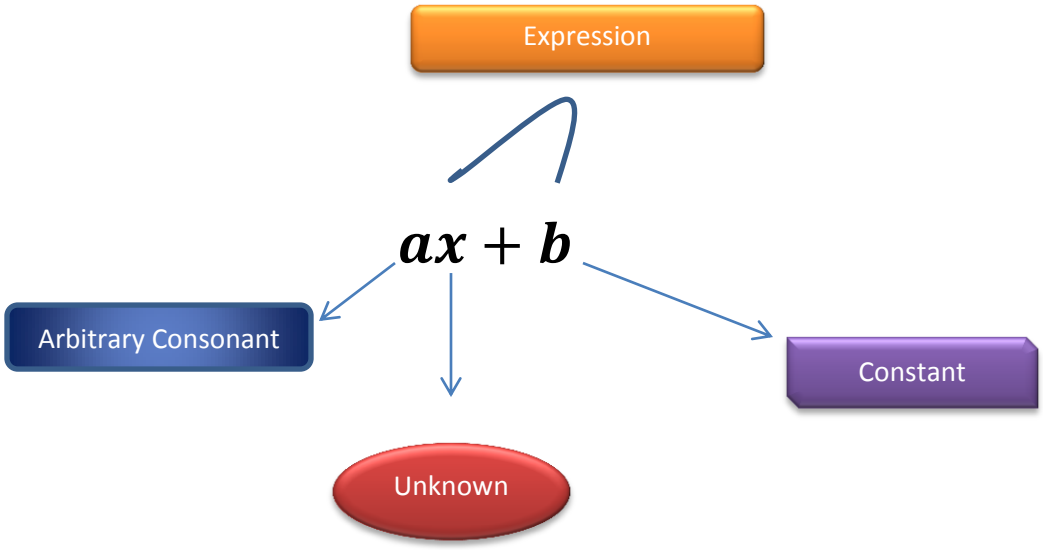


Now facilitator asked the definition of Algebraic Expression –

Participants views…..

Operation, unknown, arbitrary constant (coefficient) and constant, all these together make an Expression. That is called Algebraic Expression.

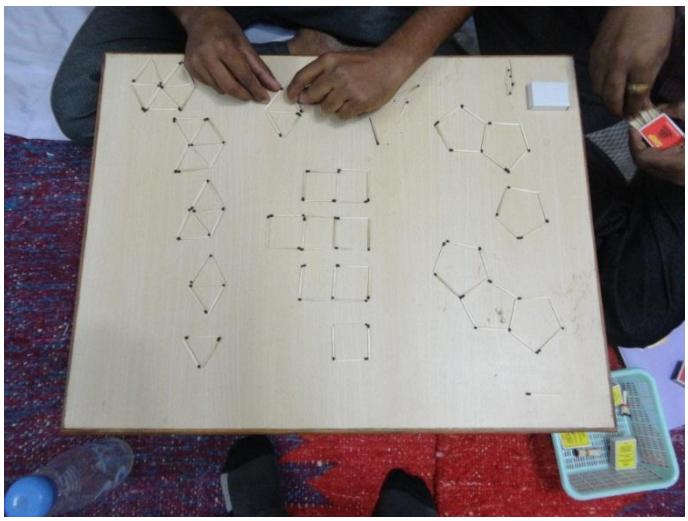
Later **facilitator elucidated** it in below way



Group Activity

For more clarification, facilitator designed the group activity and conducted it as follow;

- Participants were divided in three groups and given matchsticks.
- They were need to be prepared square, triangle and pentagon form these matchsticks
- The condition was the least no. of matchsticks to be used.



After working 20 minutes on it, they were asked two questions.

- How many matchsticks are needed for the first shape?

- How many more are needed for the next shape?

Participants prepared table on each shape and created function rule or algebraic expressions

Square

Shape number	1	2	3	4	5
Number of matchsticks	4	7	10	13	16

4 matchsticks are needed for the first shape, 3 more are needed for the next shape. After first shape equal matchsticks are needed for next shapes. It is ***generalization.***

General to specific

Function Rule - Number of matchsticks = $More\ Needed\ sticks \times Shape\ number + 1$

$= 3 \times n + 1$

$3\ n + 1$ → Algebraic Expression

So and so the table, expressions and function rules were prepared for other two shapes also.

Triangle - $2\,n + 1$

Pentagon - $4\,n + 1$

Operations (Addition and subtraction) with Algebraic expression

Facilitator asked two questions to participants and invited for made it clear explaining below example

- **Addition -**

Question - Sarita has some marbles. Ameena has 10 more than Sarita. Appu says that he has 3 more marbles than the number of marbles Sarita and Ameena together have. How do you get the number of marbles that Appu has?

Participants tried and easily made some expressions. Facilitator explained it in smooth-tongued. Such as

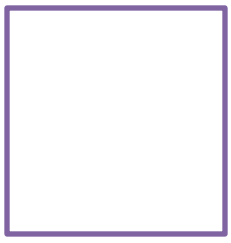
Situation (described in local language)	**Variables**	**Statement using expressions**
Sarita has some Marbles Ameena has 10 more marbles than Sarita		Ameena has $(x + 10)$ marbles.
Their brother (Appu) has 3 more marbles than both (Sarita and Ameena)	Let Sarita has x marbles.	He has $x + (x + 10) + 3$
How many marbles Appu has		

- **Subtraction –**

Question - In a garden, roses and marigolds are planted in square plots. The length of the square plot in which marigolds are planted is 3 metres greater than the length of the square plot in which roses are planted. How much bigger in area is the marigold plot than the rose plot?

Facilitator explained

There are two gardens in square shape -



x

(x+3)

Find out the difference in their area.

Area $= a^2 = a \times a$

So, $x \times x = x^2$ and $(x+3) \times (x+3) = x^2 + 6x + 9$

Difference between them

$$(x^2 + 6x + 9) - (x^2)$$

$$x^2 + 6x + 9 - x^2$$

$$\cancel{x^2} + 6x + 9 - \cancel{x^2}$$

The answer is $6x + 9$

Facilitator added ……..

The motive behind doing these exercises was to teach children creating algebraic expression (AE) and using operation (Addition and subtraction)

After elaborating both the operations, facilitator asked to all participants for making 5-5 problems, in the context of their surroundings, individually. We discussed few of them (one or two problems of each participant) and created AE. Such as

1. गायत्री के पास कुछ बर्फी के चौके थे, उसमें से कुछ चौके बाबू खा गया। बताओं गायत्री के पास कितने चौके बचे। $(x - y) = remaining\ pieces\ with\ Gayatri$
2. मांगी लाल के पास 30 कंचियां थी, पप्पू ने उसमें कुछ और कंचे उसे दिये। बताओं मांगी लाल के पास कितने कंचे है? $(30 + x) = Total\ marbles\ to\ Mangilal$
3. अमिता ने अपनी सहेली के जन्मदिन पर बाजार से कुछ रू का एक केक खरीदा और 65 रू की एक चॉकलेट खरीदी। यदि उसने कुल 215 रू खर्च किए तो बताओ केक का मूल्य क्या है? $(x + 65 = 215)$

4. रमेश के पास जितनी पुस्तके है, मोहन के पास उसकी दोगुनी पुस्तके है। कुल पुस्तकों की संख्या बताओ। $(x + 2x = total\ books)$
5. राम की मां ने कुछ रोटियां बनाई उनमें से श्याम ने 5 रोटी खाली बताईए शेष कितनी रोटियां बची? $(x - 5 = remaining\ chapatis)$

Feedback and Valedictory Session

Feedback on performance, when effective, is widely considered to be integral to learning. People learn faster and more deeply if they know what the strengths and weaknesses of their performance are and most importantly, how to improve future performance. So at last participants were asked for giving feedback on the happening of last three days.

"It was great show and inspired us that how we can improve the teaching practices through using different approaches and going in depth of content. We got source of information and linkage, even learnt how they can be used in interesting way" One of participants said.

"It was an excellent presentation from all aspect of capacity building and knowledge enhancement." Another participant pointed out.

"It was greatest experience for me by far. All the participants respected each other and worked together, it generated the health and ultimate learning atmosphere" Facilitator said.

Participants also filled the two pages format in which they need to rate the sessions and put their comments.

At last again Abhishek led the group and shared future work plan with participants.

"It was part I (basics of Algebra) another workshop will be held with you soon on part – II (Advanced Algebra)" he stated and proposed 5 methods to carry forward this engagement.

- Class Room Programs - .....(need to add something)
- Assignment – a set of assignment was given to all on which they were need to work before beginning of part – II **(For detail please see Annex – 3)**
- Mentoring and working together – participants invite the RPs whenever they teach Algebra to students so that the suitable pedagogy can be developed for students of respective schools.

- Access to Resources – We have opened the Block Activity Centre at each block. There will be library and computer with internet service (hopefully, will start from on last week of October). Any of you come and access the resources.
- Forming informal Group (Teacher forum) – Mr. Anil, Participant and member of VTF, Niwai introduced the principals of VTF.

At the end, he extended Vote of thanks to the participants and facilitators for making the consultation a successful one.

Part III

Annexures

1. Agenda

The workshop plan is as follows:

Work plan for Algebra workshop (24th to 26th September, 2012)

First day-24thSeptember, 2012			
S.No.	**Time**	**Content of sessions**	**Facilitator**
1.	10:00 AM-10:30 AM	Registration	Dinesh
2.	10:30 AM-11:00 AM	Introduction with Participants	Hanuman & Dinesh
3.	11:00 AM-11:30 AM	Agenda sharing	Hanuman & Dinesh
4.	11:30 AM-1:30 PM	I. History of Algebra • Developmental Stages in Algebra • Evolution of Algebra in different civilization	Hanuman & Ashish
5.	1:30 PM- 2:30 PM	Lunch break	
6.	2:30 PM- 2:45 PM	Warm up activity	Dinesh
7.	2:45 PM- 5:00 PM	II. What Algebra Is…? • Generalization of Arithmetical Ideas (Algebraic Thinking) III. Need, Importance & difficulties in Learning Algebra • Why Is Algebra Important to Learn…? • Difficulties in Learning Algebra	Hanuman & Ashish Ashish & Dharmendra
8.	8:00 PM- 9:00 PM	Movie	

Second day-25th September, 2012			
1.	9:00 AM-9:30 AM	Assembly	
2.	9:30 AM-10:00 AM	Recap	Hanuman
3.	10:00 AM-12:00 AM	IV. Basic Mathematics for Algebra • Integers • Fractional Numbers • Decimal Numbers • Order of Basic Operations with Numbers • Ratio & Proportions • Unit Cancellations & Conversions • Prime Factorizations • HCF & LCM • Patterns • Basic Geometrical Shapes • Powers & Scientific Notations	Dinesh & Dharmendra
4.	12:00 Noon-1:30 PM	v. Text analysis(NCERT ,SCERT & private publication) based on- • Concept of unknown • Previous knowledge • Importance of learning • Activities • Methodology & concept flow	Vinod & Dinesh
5.	1:30 PM- 2:30 PM	Lunch break	
6.	2:30 PM- 2:45 PM	Warm up activity	Vinod
7.	2:45 PM- 3:45 PM	Vi. Introduction to Algebra • Understanding of unknown using Patterns	Ashish & Dinesh

		• Application of Variables in Common rules • Expressions containing Variables - Algebraic Expression • Terms of an Expression • Algebraic Expressions - Monomial, Binomial, Trinomials & Polynomials	
8.	3:45 PM- 5:00 PM	Vii. Class room problems related to algebra • Identification of problems • Analysis of problems	Dharmendra & Hanuman
9.	8:00 PM- 9:00 PM	Movie	
Third day-26th September, 2012			
1.	9:00 AM-9:30 AM	Assembly	
2.	9:30 AM-10:00 AM	Recap	Dinesh
3.	10:00 AM-11:30 AM	Viii. Operations with Algebraic Expressions & their Properties • Addition in Algebraic Expressions • Subtraction in Algebraic Expressions	Dinesh & Vinod
4.	11:30 AM-11:45 AM	Tea Break	
5.	11:45 AM- 12:45 PM	IX. Operations with Algebraic Expressions & their Properties • Multiplication in Algebraic Expressions	Dharmendra & Hanuman

6.	12:45 PM- 1:30PM	X. Linear Equations in One Variable • What is an Equation? • Setting Up of an Equation for a given Situation & Vice-Versa • Solution of Linear Equations in One Variable	Ashish & Hanuman
7.	1:30 PM- 2:30 PM	Lunch break	
8.	2:30 PM- 2:45 PM	Warm up activity	Dharmendra
9.	2:45 PM- 4:00 PM	XI. Linear Equations in One Variable • Application of Simple Equations to Practical Situations	Ashish & Dinesh
10.	4:00 PM- 4: 30 PM	Written feedback	Hanuman & Dinesh
11.	4:30 PM- 5:00 PM	Over all consolidation & further strategy	Abhishekji

2. List of participants –

S. No.	Name of Participants	School	Block	Mobile No.
1	Ram Sahay Mali	GUPS, Ghati	Malpura	9667703917
2	Vijay Bhisht	GUPS, Sirohi	Deoli	9529254165
3	Vikram Singh	GUPS, Gadoli	Uniyara	9667041617
4	Ratan Lal Yogi	GUPS, Anawarpura	Tonk	9772828252
5	Ramratan Khatik	GUPS, Mandawar	Tonk	9680766711
6	Anil Kumar Gupta	Banasthali Vidyapith,	Niwai	9414543762
7	Ravi Jain	Banasthali Vidyapith,	Niwai	9887466548

8	Sarika Goyal	Banasthali Vidyapith,	Niwai	9887777167
9	Aamir Rahman	Regional, SSS, Tonk	Tonk	9214486444
10	Faiyyajjudin Shekh	GUPS, Bajoliya	Uniyara	9829961079
11	Ramesh Chand Vijay	GSS, Kassir	Deoli	9414841736
12	Kishan Lal Balai	GUPS, Saroli	Deoli	9829883418
13	Kalu Ram Mali	GUPS, Bapunagar	Malpura	9887795273
14	Anil Kumar	GUPS, Bileda	Malpura	9982561876

3. **Assignment –**

- चर की अवधारणा सीखने के लिए हम कक्षा कक्ष मे कौन कौन से नये तरीके इजाद कर सकते है?
- बीजगणित में सामान्यीकरण की अवधारणा के लिए ऐसे कौन कौन से नये तरीके हम कक्षा कक्ष मे इजाद कर सकते है?
- बीजगणित मे बच्चें किस किस प्रकार की गलतियॉ करते है और उनके पीछे क्या कारण हो सकते है उनकी जॉच पडताल करी जाये और उन्हे दूर करने के लिए क्या गतिविधियॉ इजाद की जा सकती है?
- ऐसी कौन कौन सी पहेलियॉ हो सकती है जिन्हे कक्षा कक्ष मे बीजगणित के परिचय के दौरान उपयोग लिया जा सकता है?
- बीजीय व्यजंको के योग एवं व्यवकलन की अवधारणा के लिए हम कक्षा कक्ष मे कौन कौन से नये तरीको को इजाद किया जा सकता है?